Quantum-confined hydrogenic impurity in a spherical quantum dot under the influence of parallel electric and magnetic fields

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A R T I C L E   I N F O

Article history:
Received 16 November 2007
Received in revised form 23 April 2008
Accepted 23 April 2008
Available online 10 June 2008

PACS:
73.21.La
32.60.+s
73.23.Hk

Keywords:
Spherical quantum dots
Stark and Zeeman effects
Complex-absorbing potential method
Tunneling
Resonances

A B S T R A C T S

A recently developed computational technique, the complex absorbing potential (CAP) method for locating complex poles of resonance states, is applied to calculate the field-induced energy shifts and widths of the ground state of a hydrogenic donor in a quantum dot (QD) subjected to parallel electric and magnetic fields. The present model demonstrates that resonances in a confined hydrogen atom lead to an anomalous behaviour due to the quantum confinement of the atom. We have studied the influence of magnetic field on the quantum-confined Stark effect. With increasing magnetic field, resonances shift to higher level and the width gets narrower with less impact on the oscillation amplitude. We will discuss our results in terms of a QD system realized in the GaAs/Ga_{1-x}Al_xAs sample.

1. Introduction

The recent advances in the modern fabrication techniques, such as molecular-beam epitaxy, liquid-phase epitaxy, and metal-organic chemical vapour deposition have made possible to grow low-dimensional systems such as quantum wires and quantum dots (QDs) with well-controlled dimension and composition, where quantum mechanical effects are manifested [1–5]. This achievement led to numerous important developments in basic semiconductor physics and device technology [6–12]. The inherent fundamental physics and the potential use of semiconductor nanostructures in designing of more efficient microelectronic devices are among the main motivations of strong ongoing interest in these highly confined systems today.

In the last few years, there has been a considerable amount of work on electronic and optical properties of quantum-well nanostructures [13–20]. Experimentally, a wide variety of QD systems have been studied including GaAs, InSb, CdSe, CdS, etc. and various spectroscopy methods have been applied to study various quantum confinement effects in QD's. QDs exhibit strongly size-dependent electrical and optical properties that might create many opportunities for scientific discovery. In extensive theoretical studies, the transport and optical properties of quantum-well structure have been considered to assess the importance of size quantization. In novel semiconductor nanodevices, carriers or impurities are often confined in quantum-well structures. By investigating electron transport in GaAs/Ga_{1-x}Al_xAs quantum wires, Sakaki [21] have shown that electrons can exhibit very high mobility because of suppression of elastic scattering by ionized impurities. Understanding the impurity states in the quantum well and QD nanostructures is an important problem in semiconductor physics. The presence of impurities in these low-dimensional quantum-well semiconductor structures and superlattices play a fundamental role in transport mechanism and optical properties of such systems because the spatial confinement effect leads to the formation of a sort of impurity band. The existence of this impurity band is expected to influence both the transport properties in the system and the optical properties associated with impurities. In most cases, irregularities in samples have indirect influence on the many-body structures, complicating the identification of the origin behind the peculiar behavior in the measured characteristics of QD's. Therefore, the study of impurity states in these low-dimensional heterostructures is an important aspect, and many theoretical and experimental works have been addressed the importance. Since Bastard's [22]
pioneering work about the binding energies of a hydrogenic impurity within an infinite potential-well structure, considerable attention has been devoted to the study of impurity states in quantum wells. Bryant [23] extensively studied hydrogenic impurity states in one-dimensional quantum-well wire. Zhu et al. [24] were the first to study the effect of hydrogenic impurity in spherical QDs. In fact, Ashoori and coworkers have found states bound to hydrogenic impurities, probably arising from Si dopant atoms in the GaAs quantum well, in their pioneering single-electron tunneling experiment [2]. It has been suggested that such impurities are the sources of pair-tunneling states, theoretically analyzed with a superimposed attractive 1/r-type potential [26].

Several authors have calculated the exciton and impurity states in two-dimensional quantum wells and superlattices [21–24], and for hydrogenic impurity levels of quantum-well wires and of QDs [21–25,27–32]. Up to now, most of the studies on the impurity atom in confined geometry are concerned with the impurity’s binding energy, the energy levels and the level stability, the density of impurity states and the optical absorption spectra [33–36]. Only few attempts have been made towards the Stark effect and Zeeman effect in impurity states in confined systems [31,32,37–42].

The main aim of this paper is to elucidate the external field effects on resonant tunneling through impurity states in QD structure. Resonant tunneling is an important aspect for QD devices. Knowledge of the resonance states in QDs is necessary for proper description and interpretation of tunneling processes in single-electron devices. Recent theoretical studies [31,32,43–49] have discussed both field-induced shifts and the field dependence of the carrier lifetime for QD systems. In actual QD devices, the effects induced by impurities or donor scattering centers may be quite pronounced. In light of promising IR detector application of QD systems and as well as of the recent realization of ideal single-photon source, it is desirable to investigate the electron tunneling rate under electric and magnetic fields. In our earlier work [32], we have demonstrated that Stark resonances in a confined hydrogen atom lead to interesting phenomena as a consequence of the quantum confinement of atom, contrary to the Stark effect on free atoms. In that case we have investigated the interplay between the confinement induced by the electric field and the spatial confinement of the QDs.

In the present work, we report our first-principles calculations of electric and magnetic-field effects on the impurity states in a spherical QD system. For convenience in calculations, the elementary model for this system is considered as a hydrogenic impurity inside a spherical QD. In this model, the electron is exposed to a constant potential in addition to the nuclear coulomb potential of the hydrogenic impurity. The QD is modeled by a spherical confinement potential of finite height. We discuss the level shift and resonant tunneling of electron bound to a hydrogenic impurity in a spherical QD under the influence of electric and magnetic fields to assess the importance of QD size quantization. The theoretical methods for such a study is challenging for the available computational tools. The existing attempts to study such systems are based on calculations in which the effect of electric field is treated by approximate techniques (i.e. perturbation theory, variational method) appropriate in the weak field limit. In such calculations, the confined states are viewed as bound states, and the field-induced broadening is ignored. However, several studies of the effects of external electric fields on the quantum-well systems have predicted both the field-induced level shifts and the field dependence of the carrier lifetime. In the literature, theoretical studies of such systems have been dealt with the phase shift method [43], the complex energy method [44, 45, 47] and the stabilization method [46]. The application of these existing techniques leads to conceptual or computational difficulties. The numerical technique used in our calculation, termed complex absorbing potential (CAP) method, has been previously used to study the Stark resonances on the ground state of a hydrogenic impurity in such a spherical QD [32]. It has been shown that the CAP method is a powerful computational tool for investigations of atomic resonances with or without the presence of external electric fields [50,51].

2. Formulation

In the effective-mass Hamiltonian of the system, with both of the external magnetic and electric fields in the direction of z-axis, is given by

$$H = H_0(\beta) - \frac{1}{r} + V(r) - Fr \cos \theta,$$

(1)

where

$$H_0(\beta) = -\frac{1}{2} \frac{d^2}{dr^2} - \frac{l(l+1)}{2r^2} + \frac{1}{2} \beta r^2 \sin^2 \theta + \beta L_z$$

(2)

is the free electron Zeeman Hamiltonian, and

$$V(r) = -V_0, \quad r < R_0$$

$$= 0, \quad r \geq R_0$$

(3)

is the confining potential, and $V_0$ is the depth of the confining potential well of radius $R_0$. $F$ is the electric field strength, $\beta$ is the magnetic field strength, and $\theta$ is the angle between the radius vector and the field. The Hamiltonian is written in dimensionless form so that all energies are measured in effective atomic mass units (1 a.u. of electric and magnetic fields (1 a.u. of electric and magnetic fields (1 a.u. $= m_e^* \beta h^2 / \mu_0 e^2$), and all distances are measured in units of effective Bohr radius ($a_0^* = h^2 / m_e^* e^2$), where $m^*$ and $e$ are the electronic effective mass and the dielectric constant, respectively, of the material used in the investigation. Thus, by introducing dielectric constant in the Hamiltonian, screening of 1/r potential is taken care of. The field strengths are measured in the effective atomic units of electric and magnetic fields (1 a.u. of electric field = $e / \mu_0 a_0^*$, and 1 a.u. of magnetic field = $h / \mu_0 a_0^*$). $L_z$ is the z-component of the angular momentum operator. Since the total Hamiltonian commutes with $L_z$, the paramagnetic term, which introduces only a global energy shift, can be considered separately.

Under the influence of the electric field, the bound states of the atom become quasi-bound, and the electrons will tunnel through the potential barrier formed by the combined effect of the Coulomb and the external fields. As a result, a bound state energy becomes complex, with the real part representing the shifted resonance position, and the imaginary part being related to the lifetime of the quasi-bound state. In other words, the Stark perturbation changes the bound states of the atomic Hamiltonian, and the electron initially confined in a potential well can always lower its potential energy by tunneling out of the well when the external field is turned on. The resonant energy spectrum of QD plays an important role in studies of nanostructures, as the resonant tunneling leads to the observed effects of fast response time and negative differential resistance, which opens up the possibilities of building high-frequency modulation devices. Applications of the external magnetic field change the resonance spectrum. Apart from the field-induced increase of the energy of the quasi-bound state, which is governed by the quadratic term of the Hamiltonian, we demonstrate that the magnetic field tends to suppress the tunneling rate of electron, in agreement with the experimental observation of Ashoori et al. [25].
3. Calculations and results

As a tool for calculating the energy and the tunneling lifetime of a resonant state, we employ the CAP method [32,50,51]. We repeat here the most essential elements of the computational scheme. The CAP method introduces a complex absorbing potential, \(-i\omega W\), to the physical Hamiltonian leading to an effective Hamiltonian

\[ H(\eta) = H - i\eta W, \]

where \(\eta\) denotes the CAP strength. In order to determine the optimal value of \(\eta\), \(H(\eta)\) is diagonalized for a number of \(\eta\) values. The resulting eigenvalues can be combined to form an \(\eta\)-trajectory in the complex energy plane. Then this \(\eta\)-trajectory is examined by computing the absolute value of \(\eta(dE/d\eta)\). The pronounced minimum of \(|\eta(dE/d\eta)|\) identifies the optimal CAP strength for the resonance state, leading to the complex resonance energy

\[ E_{\text{res}} = E_t - i\Gamma, \]

where \(E_t\) is the resonance energy and \(\Gamma/2\), the resonance width (related to the tunneling lifetime \(\tau = 2\hbar\Gamma\)).

The computational tool employed in this study is the general-ized variational method. The technique is based on forming trial wave functions from a linear combination of independent functions \(\phi_k\), as

\[ \psi = \sum c_k \phi_k. \]

The Schrödinger equation for the Hamiltonian is then reduced to give the following matrix equation

\[ H(\eta)\psi = \varepsilon(\eta)\psi, \]

where \(H(\eta)\) is the Hamiltonian matrix, \(\varepsilon\) is the overlap matrix, \(\psi\) and \(\varepsilon\) are the corresponding eigenvectors and eigenvalues, respectively. Moreover, the calculated eigenvalues approach to the exact eigenvalues as the number of basis functions increases. In order to perform linear variational calculations we have chosen the orthonormal Laguerre-type basis

\[ \psi_{\text{lim}} = C_{\text{lim}}\exp(-r)\sum_{m=0}^{l_{\text{max}}} (-1)^{l+m} r^{l+m} Y_{lm}(\theta, \phi), \]

where \(n = 0,1,2,\ldots, N-1\), and \(C_{\text{lim}}\) is the normalization constant. \(N\) is the number of basis functions per \(l\) value. Calculations have been performed as a function of \(N\) and \(l_{\text{max}}\), with \(l_{\text{max}}\) being the maximum angular momentum \(l\) used in a given calculation, in spherical coordinates, \(H(\eta)\) in the absence of an external field is a block diagonal matrix in \(l\). When the fields are turned on, the block with angular momentum \(l\) is connected to the blocks with angular momentum \((l \pm 1)\) and \((l \pm 2)\), for external electric field and magnetic field, respectively. For practical calculations, each block is taken of dimension \(N\) and the number of blocks has been restricted to \((l_{\text{max}}+1)\). The matrix size is thus \([N(l_{\text{max}}+1)]^2\) in the present investigation.

To obtain the spectral distribution of the effective Hamiltonian we perform calculations with \(N = 30\) and \(l_{\text{max}} = 29\). \(\lambda\) is taken to be \(2\). The choice of CAP is motivated by our previous attempts to establish the present method as a simple alternative method with a view to making further progress in the area of resonance calculations for many electron systems, and it is made accordingly as \(W(r) = p^2\) [50]. Complex eigenvalues problems with maximum size \(900 \times 900\) are then solved, and the complex eigenvalues of \(H(\eta)\), those representing the resonance states are found to have \(\eta\) dependence. We observe that the optimal CAP strength \(\eta\)'s are sufficiently small (\(\approx 10^{-9} - 10^{-11}\)) to yield the good estimate of the complex resonance energy. Convergence of eigenvalues has been studied for each fixed value of the dot size as a function of \(\eta\), \(\lambda\), and \(N\). The value of \(l_{\text{max}}\) has been fixed at \((N-1)\) in the present calculations. The results are summarized in tables and figures.

We focus our attention on the lowest 1s state only. In our previous paper [32] we have studied the present model to study the quantum-confined Stark resonances. We observe that the resonant energy is not a monotonically decreasing function of the dot radius, whereas in the field-free case, the ground state energy eigenvalue of the confined atom monotonically decreases as the radius is increased. While the numerical results for the pure electric-field effect on the hydrogenic impurity in a spherical QD were published in Ref. [32] earlier, the underlying physics was not thoroughly discussed. Here, we take the opportunity to provide a new and in-depth discussion (than that given in Ref. [32]) on the underlying physics behind the oscillatory behavior of the resonance eigenvalue for changing size of the QD. Fig. 1 shows the effective potential (which is the combined Coulomb potential and the QD confinement) of the system when the external electric field is not on. The application of the external electric field changes the potential well surrounding the hydrogenic impurity. A sketch of the expected configuration is shown in Fig. 2.

Under the influence of an external electric field, and when the dot radius is increased, the resonant energy level is first decreased

- \[ V = \begin{cases} V_0 & r \leq R_0 \\ 0 & r > R_0 \end{cases} \]

Fig. 1. Potential energy profile for a hydrogenic impurity in a single quantum well of depth \(V_0\) and radius \(R_0\).

- \[ V = -\frac{1}{r} F \cdot z \]

Fig. 2. Potential energy profile for a hydrogenic impurity in a single quantum well of depth \(V_0\) and radius \(R_0\) subject to an external electric field \(F\).
to reach a minimum, and subsequently increases to another value as the dot radius is further increased. The oscillation continues until the amplitude of the oscillation gradually dies out at a considerable large value of the dot radius. We also find that the oscillating behavior is more pronounced in the behavior of resonant tunneling. In other words, the external electric field introduces an oscillation for the resonant states. This oscillating behavior cannot be explained classically, which is due to the fact that quantum mechanical tunneling does not make the classically bound states metastable. Upon application of the electric field resonances develop out of the bound states, but the oscillation of the resonance structure develops out of interference effect from the potential structure due to the external field and the confinement geometry of the dot, as discussed elsewhere [32]. We now explain the oscillatory phenomenon qualitatively as following: In the absence of the QD, a hydrogen atom under the influence of external DC electric field has been studied extensively in the literature (see Refs. [52,53] for example). The solutions for the ground state wave function for the hydrogen atom in electric fields are expressed in terms of Airy functions, which by themselves have oscillatory behaviors. Next, consider the situation when the hydrogen impurity is placed at the center of the dot, and hence it is confined by an extra potential of finite height due to the QD confinement (see Eq. (3) and Fig. 1). Now when the system is under the influence of an external electric field (see Fig. 2), and when a node or anti-node of the Airy functions lie at the edges of the QD, the ground state wave function will experience constructive interference or destructive interference effects. As a result, the autoionization lifetime (tunneling of the electron) will be shortened or prolonged, leading to the oscillatory behaviors of the resonance width, as revealed in our recent [32] and the present calculations.

Another aim of the present work is to study the behavior of resonance spectra of a hydrogenic impurity located at the center of a QD subjected to an external magnetic field, and to the combined effect of external electric and magnetic fields in parallel directions. We should mention that the results for the combined electric- and magnetic-field effects on the hydrogenic impurity in a spherical QD are new calculations in the literature, to the best of our knowledge. Before discussing the magnetic field dependence on resonance spectra, it is useful to review the magnetic-field effect on the ground state energy of the confined impurity atom. Fig. 3 shows the energy eigenvalues of a hydrogenic impurity associated with the lowest 1s state in a QD heterostructure for different dot radii. In Fig. 4 we plot the ground state energy versus dot radius for different values of magnetic field. Our results in the zero magnetic field case (Fig. 3) agree with the previous other results [27] that the energy eigenvalue of the confined hydrogen atom monotonically decreases as the radius is increased. As the radial confinement is relaxed, i.e. as the dot radius becomes larger, the ground state energy converges asymptotically to appropriate bulk values. Influence of the magnetic field is simply to push the energy level upward in accordance with the quadratic term present in the Hamiltonian. As discussed before, the Stark resonances in a confined hydrogen atom lead to interesting oscillation behavior due to the quantum confinement of the atom. In order to investigate the magnetic-field effect on the quantum-confine Stark effect, we have also calculated the complex eigenvalues representing the shifted and broadened 1s resonance state of the confined hydrogen atom at various dot sizes for different magnetic field strength. The numerical values of resonance energy and width as obtained from our calculations are summarized in Tables 1–4. In Fig. 5, we plot the resonance energy at $F=0.10\, \text{a.u.}^*$ as a function of dot radius for different magnetic field strengths. The confining potential is taken as $V_0=0.05\, \text{a.u.}^*$. We observe that the magnetic field reduces the Stark shift and the oscillating behavior is very much likely to be unaffected by the increase of the magnetic field strength. Similar
behavior is also observed in the resonance width as depicted in Fig. 6. The increasing magnetic field only reduces the resonance width indicating longer tunneling lifetime. Thus, the presence of magnetic field imposes an additional mechanism of confinement for the impurity atom and the competing effects of field-induced confinements and the spatial confinement of the QD modify resonance states. The anomalous resonant tunneling is highly unlikely to be predicted, but a rigorous study of the actual potential structure might explain this unusual behavior. Our study suggests that the size of the QD and the strength of the confinement potential control the resonance structure of the confined atom. Figs. 7 and 8 exhibit the resonance structure of

Table 2
Complex eigenvalues representing the shifted and broadened ground state of a confined hydrogen atom in a QD as function of dot radii for $V_0 = 0.05 \text{ a.u.*}$, $F = 0.075 \text{ a.u.*}$ and $\beta = 0.075 \text{ a.u.*}$

<table>
<thead>
<tr>
<th>$R_0 (\text{a}_0^\ast)$</th>
<th>$E_i (\text{a.u.})$</th>
<th>$\Gamma/2 (\text{a.u.})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.516331</td>
<td>$5.51 \times 10^{-3}$</td>
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<tr>
<td>1.0</td>
<td>-0.530864</td>
<td>$3.38 \times 10^{-3}$</td>
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<tr>
<td>2.0</td>
<td>-0.551563</td>
<td>$2.26 \times 10^{-3}$</td>
</tr>
<tr>
<td>3.0</td>
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<tr>
<td>4.0</td>
<td>-0.564739</td>
<td>$2.87 \times 10^{-3}$</td>
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<tr>
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<td>$3.60 \times 10^{-3}$</td>
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<tr>
<td>7.5</td>
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<td>$5.68 \times 10^{-3}$</td>
</tr>
<tr>
<td>10.0</td>
<td>-0.566571</td>
<td>$5.06 \times 10^{-3}$</td>
</tr>
<tr>
<td>12.5</td>
<td>-0.566440</td>
<td>$4.98 \times 10^{-3}$</td>
</tr>
<tr>
<td>15.0</td>
<td>-0.566421</td>
<td>$5.07 \times 10^{-3}$</td>
</tr>
<tr>
<td>20.0</td>
<td>-0.566345</td>
<td>$5.11 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 3
Complex eigenvalues representing the shifted and broadened ground state of a confined hydrogen atom in a QD as function of dot radii for $V_0 = 0.05 \text{ a.u.*}$, $F = 0.075 \text{ a.u.*}$ and $\beta = 0.1 \text{ a.u.*}$

<table>
<thead>
<tr>
<th>$R_0 (\text{a}_0^\ast)$</th>
<th>$E_i (\text{a.u.})$</th>
<th>$\Gamma/2 (\text{a.u.})$</th>
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<tbody>
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<tr>
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<td>$8.86 \times 10^{-4}$</td>
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<tr>
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<td>-0.558832</td>
<td>$1.01 \times 10^{-3}$</td>
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<tr>
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<td>$1.02 \times 10^{-3}$</td>
</tr>
<tr>
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<td>$9.80 \times 10^{-4}$</td>
</tr>
<tr>
<td>20.0</td>
<td>-0.558927</td>
<td>$9.93 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Figure 5. Quantum size effect on the Stark width of the lowest 1s resonating state of confined hydrogen atom at $F = 0.1 \text{ a.u.}$ for different magnetic field strengths.

Figure 6. Quantum size effect on the Stark width of the lowest 1s resonating state of confined hydrogen atom at $F = 0.1 \text{ a.u.}$ for different magnetic field strengths.

Figure 7. Quantum size effect on the field-induced energy shift of the lowest 1s resonating state of confined hydrogen atom at $F = 0.075 \text{ a.u.}$ for different magnetic field strengths.
the impurity atom for $V_0 = 0.05 \text{ a.u.}^*$ at $F = 0.075 \text{ a.u.}^*$. We observe that resonance energy of the lowest state now increases gradually to the asymptotic value with little fluctuation in its behavior, but resonant tunneling considerably fluctuates with the size of the dot. Figs. 9 and 10 show our results for the changes of the resonance energies and widths, respectively, for the ground state of confined hydrogen atom as functions of $\beta$ for an electric field strength of 0.10 a.u.* when different values of dot radii are used. We observe that the width starts to decrease and tries to neutralize the effect of the electric field with increasing magnetic field strength. It shows the possible stabilization of the autoionization process of the confined atom under the influence of the magnetic field. We now explain such phenomenon as following: when the external magnetic field is turned on, the electron is confined to a region further away from the potential barrier. Therefore, it would require a longer time to tunnel out of the barrier, resulting in decrease of the width for increasing magnetic field strength. We believe that the results we have obtained represent a major advance in the field, and that hopefully our work will stimulate further investigations on such interesting phenomenon.

**4. Discussion**

One of the crucial problems in semiconductor physics and particularly in semiconductor nanostructures is the presence of impurities, which play a fundamental role in transport mechanisms at low temperature. Their associated electronic bound states are hydrogen-like states similar to those found in the impurity problem in two-dimensional semiconductor structures [22]. In fact, shallow donor hydrogen complex has been reported previously in hydrogenated melt-grown (HB) Si-doped GaAs [54]. Leitch et al. [55] have also reported hydrogen-related defects in passivated n-type GaAs grown by metal-organic vapor-phase epitaxy (MOVPE). They also presented evidence to show that hydrogen is responsible for the metastable levels in MOVPE-grown GaAs reported by Buchwald and coworkers [56]. Therefore, hydrogen can be incorporated in the GaAs during MOVPE and may have important implications when considering the growth of device structures by MOVPE. In the context of our present work, we should point out that the performance of a semiconductor device is dictated by shallow dopants. Shallow dopants like phosphorus and boron alter the conductivity of bulk silicon by several orders of magnitude. Deep defects, on the other hand, are known to degrade device performance. Relevance of the role of shallow donors in semiconductor QDs, which are essentially zero-dimensional structures, thus cannot be overestimated.

As a particular realization of the present model one can consider the structure, build up in GaAs/Ga$_{1-x}$Al$_x$As with properly chosen composition of Ga and Al atoms. The impurity state is presumably due to substitutional or interstitial Si atom migrated through the relatively thin space layer. For the present calculation, we have modeled the donor impurity as a hydrogenic impurity located at the centre of the QD and screened by the bulk static dielectric constant of GaAs. The electron bound states and the associated energies have been obtained by solving the effective-mass Schrödinger equation. Since the impurity binding energies (order of meV) are much smaller than the band gap of GaAs (1.4 eV), the effective-mass approximation is appropriate to treat the problem. The value of the confining potential can be fixed by the Al concentration $x$ in GaAs/ Ga$_{1-x}$Al$_x$As through the expression for the energy band-gap discontinuity $\Delta E_g = 1.04x + 0.47x^2$. We particularly emphasize on the resonant tunneling through impurity-related defect states. This makes accessible the physics of the widths of the QD states, electron transfer times, and scattering contributions in transport.
through submicrometer-diameter resonant tunneling diodes. All the results are presented in effective atomic units (a.u.). Using $m^* = 0.067m_e$ and $\epsilon = 13.13$ for donor impurities in a GaAs QD, these units correspond to length unit $a_0^* = 103.67\, \text{Å}$, 1 a.u. of energy = 10.3 meV, 1 a.u. of electric field = 10.2 kV/cm, and 1 a.u. of magnetic field = 12.2 T. For our present calculation in a real sample, we have studied the effect of field-induced energy shift and resonant tunneling for a QD of size 1 $a_0^*$ ($\approx 100\, \text{Å}$ approximately). In the present study, the onset of the field regime for a finite potential well surrounding the impurity can roughly be defined as $F > F_c$; where $F_c$ corresponds to the value of the applied field for which the top of the well is pulled down by the field to the level of the zero field bound state. The approximate condition for this to occur is $F_R_0 = V_0$. This calculation gives $F_c \approx 500\, \text{V/cm}$ for $V_0 \sim 0.5\, \text{meV}$. In moderate fields ($F \ll F_c$) the Stark should nevertheless be observable since, the tunneling rate should be small compared to the spectroscopic observable time. For fields above $F_c$, the tunneling rate will be high and this is confirmed by studying the resonance widths, which are evident from figures and tables. For resonance energy, it becomes more negative as the applied field is increased but the drastic increase in width at very high fields means that it is doubtful in the context of spectroscopic studies that resonance position has any physical meaning for $F > F_0$, where $F_0$ is the field for which tunneling barrier to ionization disappears. The approximate condition for this to occur is $F_R_0 = E_0$, where $E_0$ is the zero field binding energy. So it appears unlikely to observe tunneling rate experimentally for fields greater than $5.5\, \text{kV/cm}$ in the present study. Here we focus on the effect of varying electric and magnetic fields on the resonant tunneling from such a real sample. Considering the applied electric field to be $1.02\, \text{kV/cm}$, we estimate the tunneling lifetime is of the order 11.45 ps and with the application of the magnetic field of strength $1.22\, \text{T}$, the resonance lifetime increases to a value 16.66 ps. Since the resonant tunneling determines the dark current, it is desirable to assess the tunneling rate for promising applications of QD systems. Our study indicates that the local electric field may cause undesirable device properties through resonant tunneling and the stability of the confined atom can be controlled by the size of the QD and by the application of external magnetic field. One can therefore exploit the interplay between electric field strength, magnetic field strength, and the size of QDs in device fabrication, and possible device applications of field-induced tunneling.

Next, we comment on the CAP method employed in our present work as related to the other method, the complex-scaling (CS) method (or the complex-coordinate rotation) method [57] that is often used in resonances investigations in atomic physics. In theory, the CS method is applicable to a potential that is dilatation analytic. In our present model calculation, the potential in no longer dilatation analytic as the functional values of the potential and their derivatives are not smooth functions at the edge of the QD. In principle, such a shortcoming can be overcome by using an exterior complex-scaling (ECS) method, a modified version of the CS method. However, the computational aspect of the ECS is somewhat more involved, and we prefer to use the present CAP method as the integrals are strict forward when we use the orthonormal Laguerre-type basis functions on the quantum-confined hydrogenic potential.

Next, we now comment on the strength of the quantum confinement used in our present model problem. We would like to point out that even for a seeming weak confinement: our findings indicate that the effect could be quite large. In our example, the confinement is about 10% (0.5 meV) of the 5 meV impurity binding energy in GaAs. Very often in practical systems, the confinement in a QD could be quite large (see Ref. [58], for example). Hence our model problem may be viewed as an impurity centered at a dot with a weak background potential well. Nevertheless, our findings indicate that even though the confinement is seemingly weak; the electric- and magnetic-field effects could still be quite pronounced, depending on the size of the dot, especially for the resonance width, which is related to the tunneling lifetime. We hope our findings will stimulate further studies on such intrigue phenomenon. Finally, we should also mention that in our model treatment, we have not considered the particle–particle interaction that may be important in real systems. A study of such effect is outside the scope of our present investigation.

5. Conclusion

We have investigated field-induced energy shift and resonant tunneling from an impurity atom confined in a spherical QD. It is apparent from the results that resonant tunneling is very much sensitive of the dot size and as well as the confinement potential. It is demonstrated that, at experimentally attainable fields (0.5–5.5 kV/cm), the tunneling widths are in fact quite large, and that qualitatively new phenomena occur. This fact is very important for a correct description of impurity-related absorption and photoluminescence experiments. Doping in semiconductor has significant technological impact. As transistors and integrated circuits decrease in size, the physical properties of the devices are becoming sensitive to the actual configuration of impurities. Therefore, resonance oscillation will place heavy burdens on the device fabrication and precise control is necessary over the fabrication. Such control can presumably be achieved by various band-engineering procedures that can precisely fix the strength of the confinement potential and the size of the device in order to avoid any unnecessary effect. Moreover, our present results show the usefulness of the CAP method in studying the resonant tunneling phenomena in QD structures. Working with the finite Laguerre basis set, we are able to obtain stable results of energy positions and lifetimes for resonance states. The method is very efficient and computer time saving compared to the other standard approaches. It thus constitutes a useful tool for the investigation of energy structure of low-dimensional systems.

Acknowledgements

This work is supported by the National Research Council of Taiwan, ROC. Y.K.H. would like to thank Professor A.R.P. Rau for valuable discussion about the electric field effect on a free hydrogen atom.

References
